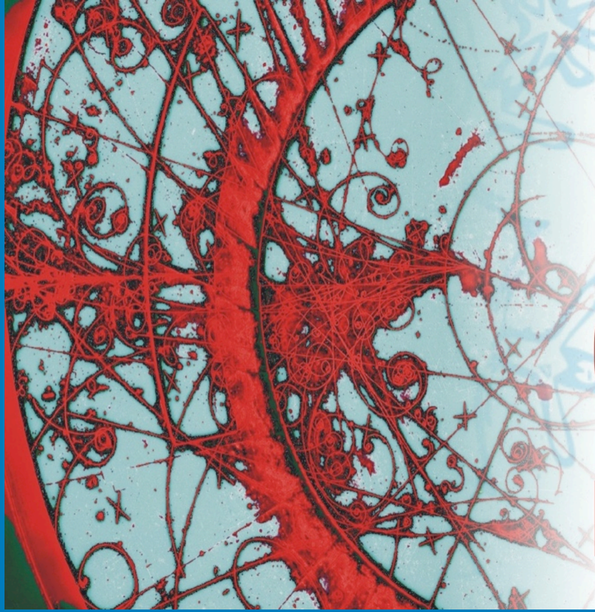


Дубненская международная школа современной теоретической физики

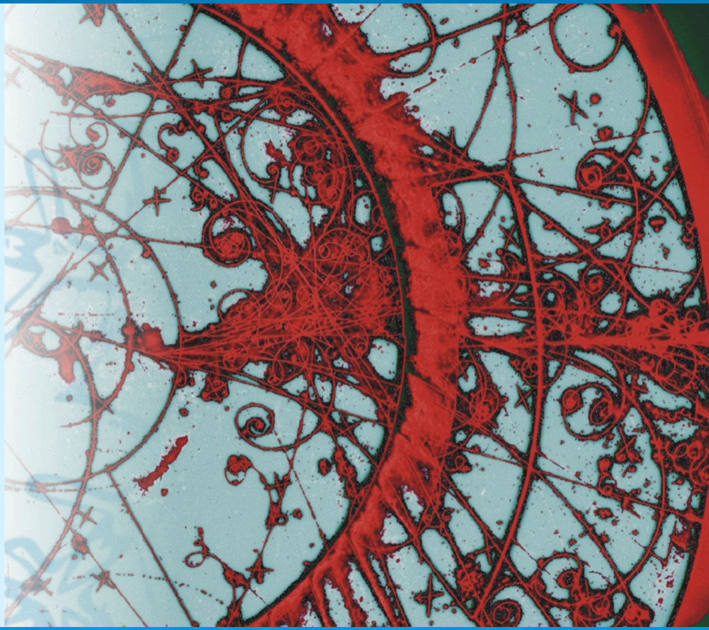
X Зимняя школа по теоретической физике

# ФИЗИКА НА БОЛЬШОМ АДРОННОМ КОЛЛАЙДЕРЕ

30 января - 6 февраля, 2012
ЛТФ ОИЯИ, Дубна, Россия

# The Standard Model of Fundamental Interactions

Dmitry Kazakov (JINR)



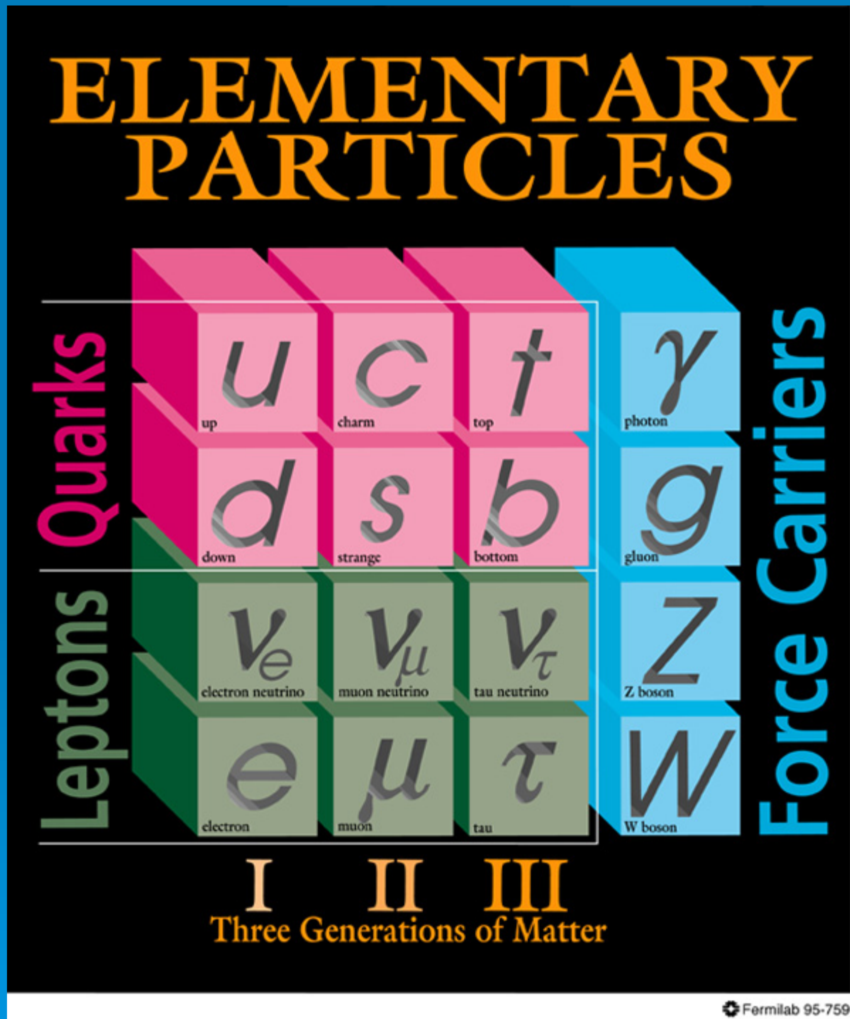
# The Standard Model

SU(3

SU(

U(

Particles




H

The Higgs boson

## Forces

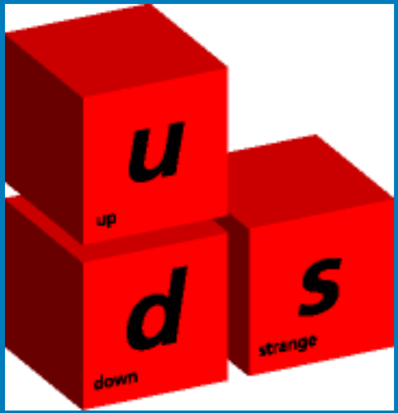
Electromagnetic

Strong

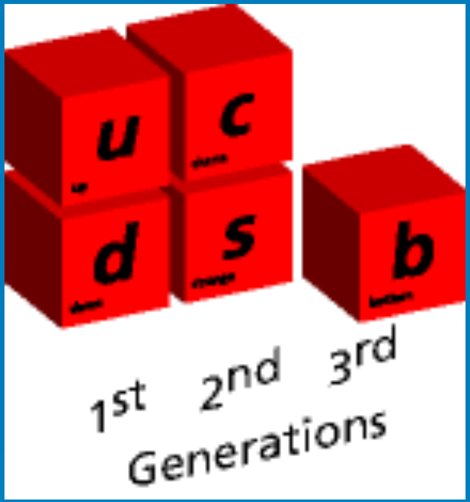
Weak

Gravity

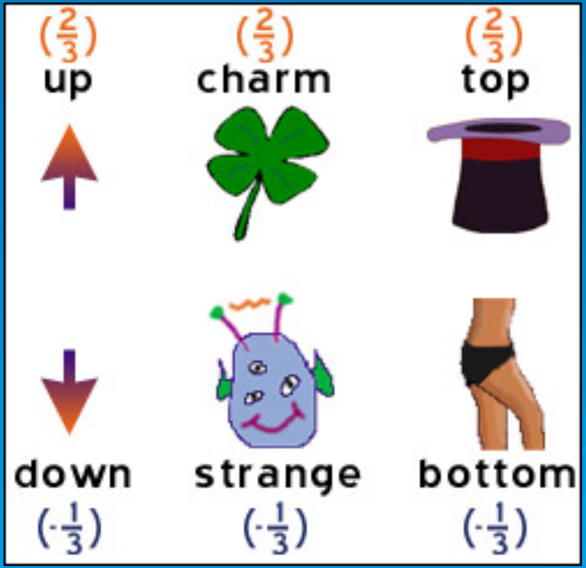
# Quarks – “the building blocks of the Universe”



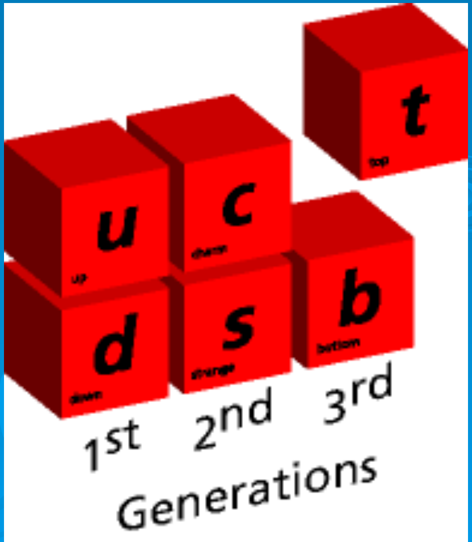

Charm came as surprise but completed the picture


The number of quarks increased with discoveries of new particles and have reached 6




For unknown reasons Nature created 3 copies (generations) of quarks and leptons



# Discovery History

| | | |
|---|---|---|
| u | c 1974 | t 1995 |
| d | s 1947 | b 1977 |

six quarks

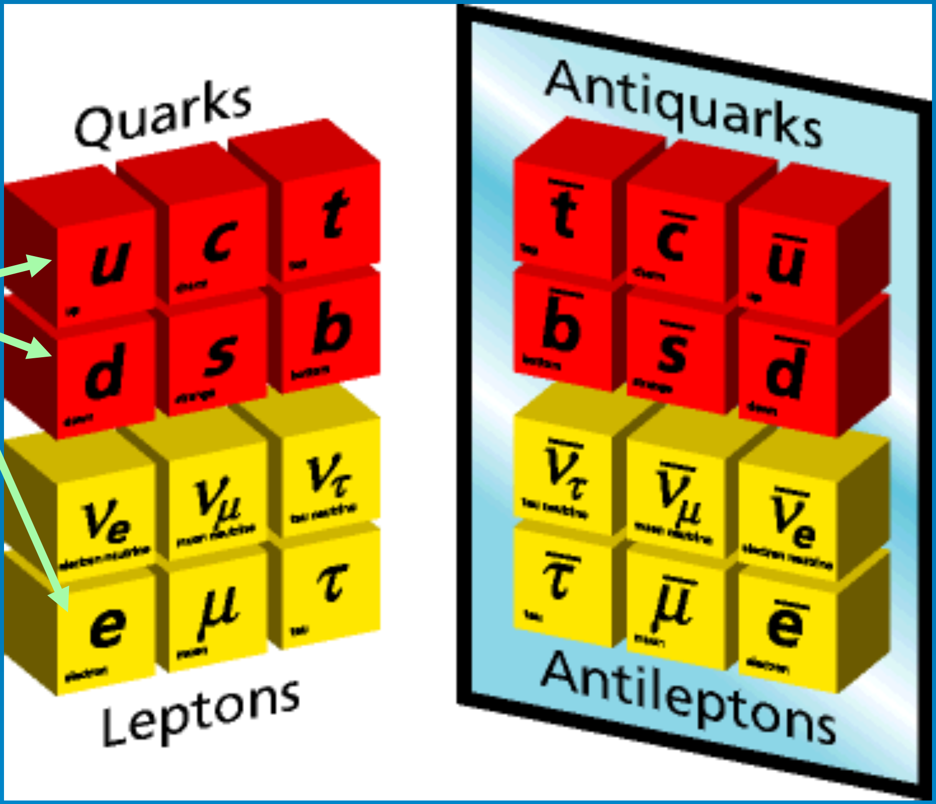
| | | |
|---|---|---|
| $n_e$ 1956 | $n_m$ 1963 | $n_t$ 2000 |
| e 1895 | m 1936 | t 1975 |

six leptons

Now we have a beautiful pattern of three pairs of quarks and three pairs of leptons. They are shown here with their year of discovery.

# Matter and Antimatter

The first generation is what we are made of

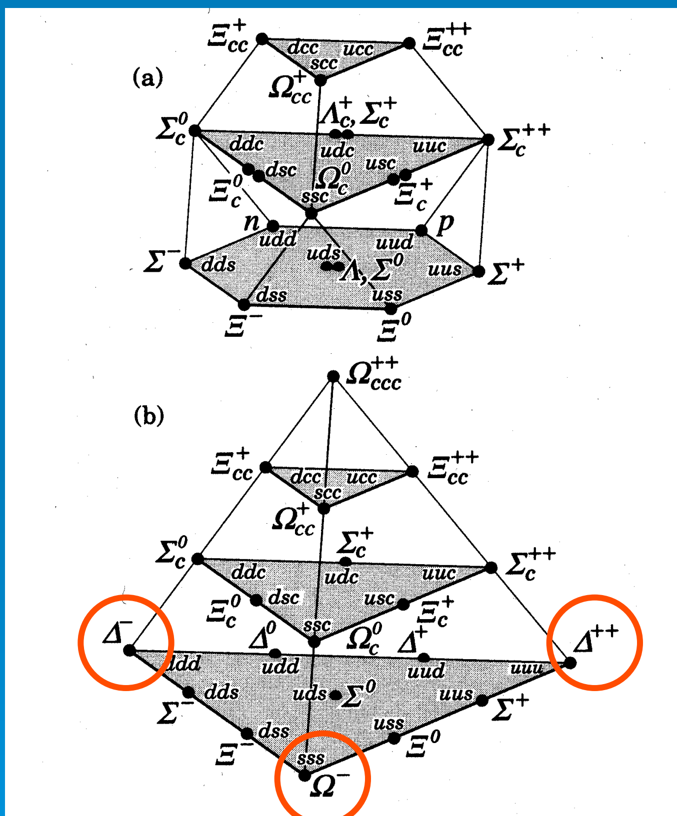



Antimatter was created together with matter during the "Big bang"

Antiparticles are created at accelerators in ensemble with particles but the visible Universe does not contain antimatter

# Quark's Colour

Baryons are "made" of quarks




$$\Delta^{-}(d\uparrow d\uparrow d\uparrow)$$

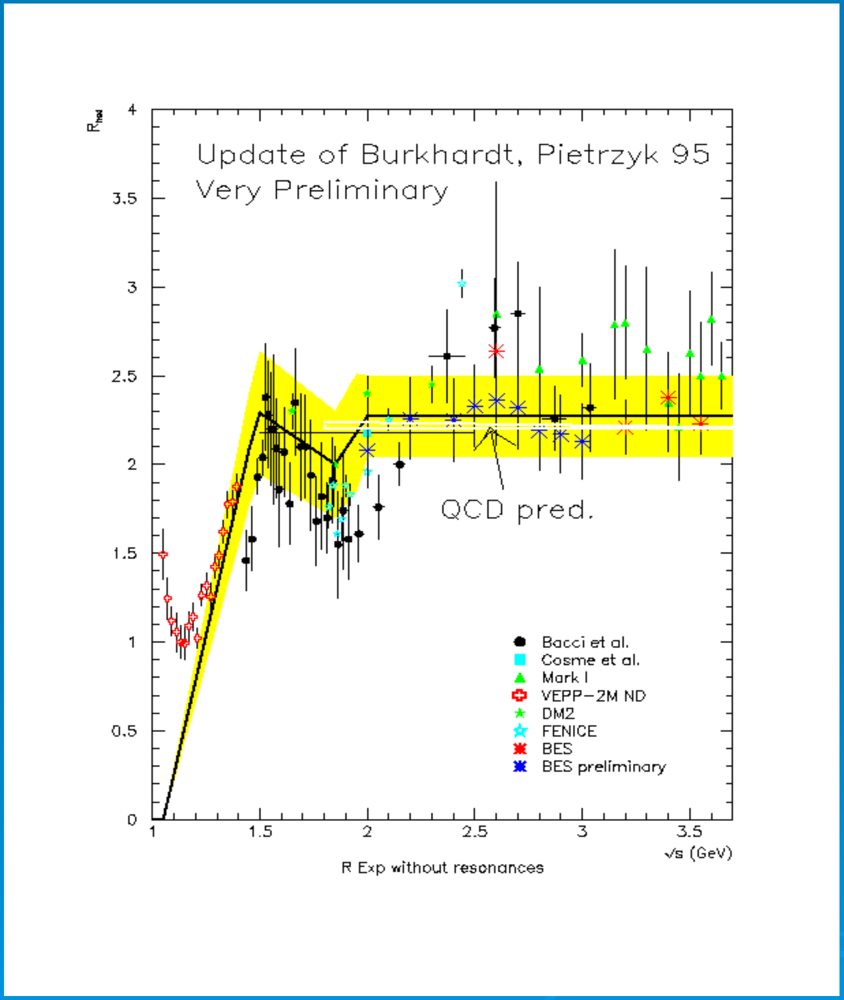
$$\Omega^{-}(s\uparrow s\uparrow s\uparrow) \qquad ?$$

$$\Delta^{++}(u\uparrow u\uparrow u\uparrow)$$

To avoid Pauli principle veto one can antisymmetrize the wave function introducing a new quantum number - "colour", so that

$$\Delta^{-} = \varepsilon^{ijk}(d_i\uparrow d_j\uparrow d_k\uparrow)$$

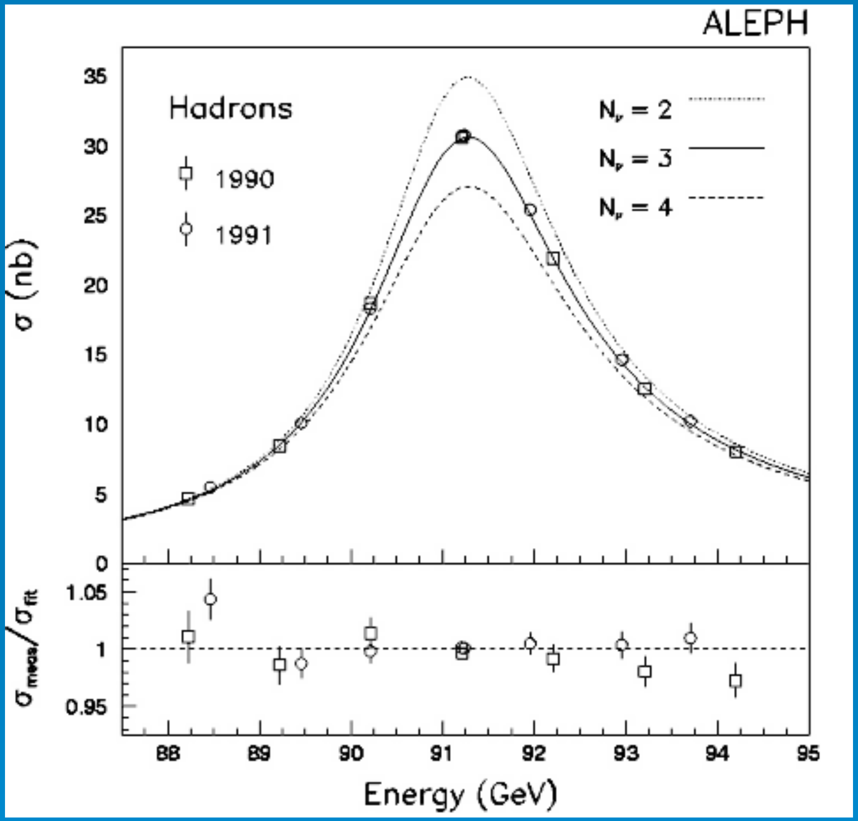
# The Number of Colours




- The x-section of electron-positron annihilation into hadrons is proportional to the number of quark colours. The fit to experimental data at various colliders at different energies gives

$$N_c = 3.06 \pm 0.10$$

# The Number of Generations




$N_g = 2.982 \pm 0.013$

- Z-line shape obtained at LEP depends on the number of flavours and gives the number of (light) neutrinos or (generations) of the Standard Model

# Quantum Numbers of Matter

- Quarks

$$Q_L = \begin{pmatrix} up \\ down \end{pmatrix}_L$$

$$U_R = up_R$$

$$D_R = down_R$$

| | SU(3)$_c$ | SU(2)$_L$ | U$_Y$(1) |
|---|---|---|---|
| $Q_L$ | 3 | 2 | $1/3$ |
| $U_R$ | 3 | 1 | $4/3$ |
| $D_R$ | 3 | 1 | $-2/3$ |

triplets

doublets

singlets

V-A currents in weak interactions

- Leptons

$$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{matrix} \frac{1}{2} \\ -\frac{1}{2} \end{matrix} \; T_3$$

$$N_R = \nu_R \; ? \qquad 0 \; T_3$$

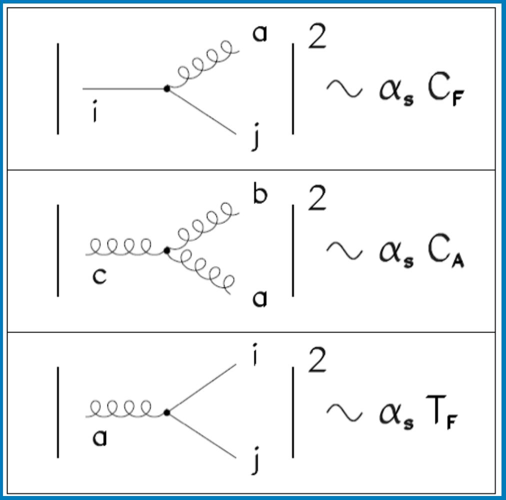
$$E_R = e_R \qquad 0$$

| | SU(3)$_c$ | SU(2)$_L$ | U$_Y$(1) |
|---|---|---|---|
| $L_L$ | 1 | 2 | $-1$ |
| $N_R$ | 1 | 1 | 0 |
| $E_R$ | 1 | 1 | $-2$ |

Electric charge

$$Q = T_3 + Y/2$$

# The group structure of the SM



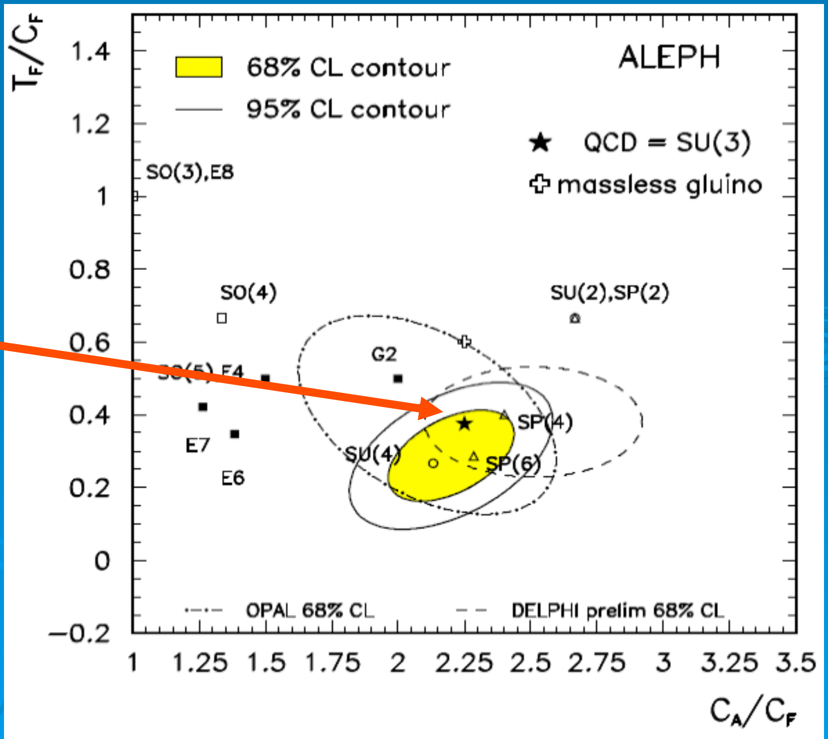

$$\sum_{a=1}^{N_A} \left(T^a T^{\dagger a}\right)_{ij} = \delta_{ij} C_F \quad , \quad \sum_{i,j=1}^{N_F} T^a_{ij} T^{\dagger b}_{ji} = \delta^{ab} T_F \quad , \quad \sum_{a,b=1}^{N_A} f^{abc} f^{*abd} = \delta^{cd} C_A$$

Casimir Operators

For SU(N)

$$C_A = N_C \quad , \quad C_F = \frac{N_C^2 - 1}{2N_C} \quad , \quad T_F = 1/2$$

QCD analysis definitely singles out the SU(3) group as the symmetry group of strong interactions



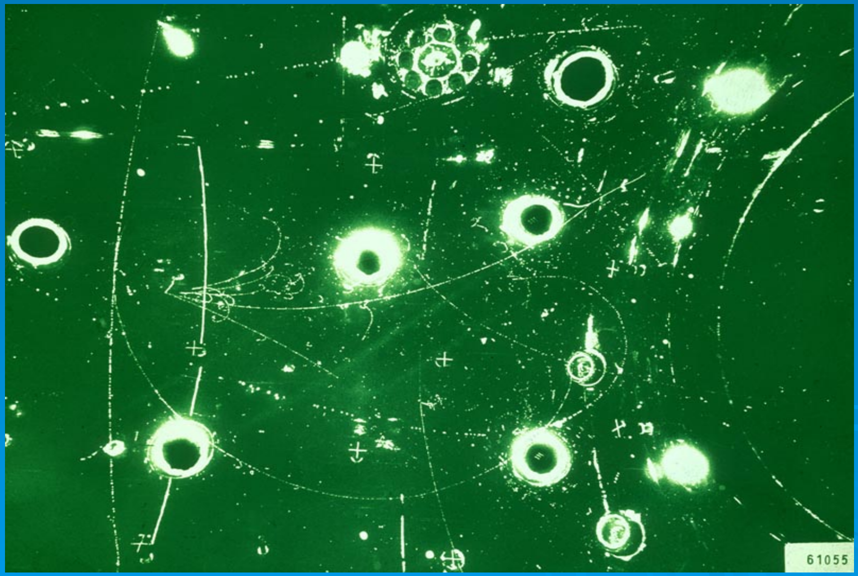
# Electro-weak sector of the SM

SU(2) x U(1) versus O(3)

3 gauge bosons 1 gauge boson 3 gauge bosons

After spontaneous symmetry breaking one has

| | |
|---|---|
| 3 massive gauge bosons ($W^+$ , $W^-$ , $Z^0$) and 1 massless (γ) | 2 massive gauge bosons ($W^+$ , $W^-$ ) and 1 massless (γ) |




- Discovery of neutral currents was a crucial test of the gauge model of weak interactions at CERN in 1973
- The heavy photon gives the neutral current without flavour violation

# Gauge Invariance

Gauge transformation $\psi_i(x) \to \hat{U}_{ij}(x)\psi_j = \exp[i\alpha^a(x)T^a_{ij}]\psi_j$ $\quad a = 1, 2, \ldots, N$

$\bar{\psi}_i(x) \to \bar{\psi}_j \hat{U}^+_{ji}(x)$ $\quad$ $\hat{U}$: matrix, $\alpha^a$: parameter, $T^a_{ij}$: matrix $\quad \hat{U}^+\hat{U} = 1$

Fermion Kinetic term $i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) \to i\bar{\psi}(x)\hat{U}^+(x)\gamma^\mu\partial_\mu\left(\hat{U}(x)\psi(x)\right)$

$$= i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) + \underline{\bar{\psi}(x)\gamma^\mu\hat{U}^+(x)\partial_\mu\hat{U}(x)\psi(x)}$$

Covariant derivative $\partial_\mu \to D_\mu = \partial_\mu I + gA^a_\mu T^a = \partial_\mu \hat{I} + g\hat{A}_\mu \leftarrow$ Gauge field

$$\hat{A}_\mu(x) \to \hat{U}(x)\hat{A}_\mu(x)\hat{U}^+(x) - \tfrac{1}{g}\partial_\mu\hat{U}(x)\hat{U}^+(x) \longrightarrow D_\mu\psi(x) \to \hat{U}(x)D_\mu\psi(x)$$

Gauge invariant kinetic term $\boxed{i\bar{\psi}(x)\gamma^\mu D_\mu\psi(x)}$

$$[D_\mu, D_\nu] = g\hat{G}_{\mu\nu} = g\left(\partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu + g[\hat{A}_\mu, \hat{A}_\nu]\right) \quad \hat{G}_{\mu\nu}(x) \to \hat{U}(x)\hat{G}_{\mu\nu}(x)\hat{U}^+(x)$$

Gauge field kinetic term $\boxed{-\tfrac{1}{4}Tr\ \hat{G}_{\mu\nu}\hat{G}^{\mu\nu}}$ $\quad$ Field strength tensor

# Lagrangian of the SM

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$$

$$L = L_{gauge} + L_{Yukawa} + L_{Higgs}$$

$$L_{gauge} = -\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \tfrac{1}{4} W^i_{\mu\nu} W^i_{\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B_{\mu\nu}$$
$$+ i\overline{L}_\alpha \gamma^\mu D_\mu L_\alpha + i\overline{Q}_\alpha \gamma^\mu D_\mu Q_\alpha + i\overline{E}_\alpha \gamma^\mu D_\mu E_\alpha$$
$$+ i\overline{U}_\alpha \gamma^\mu D_\mu U_\alpha + i\overline{D}_\alpha \gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger (D_\mu H)$$

$$L_{Yukawa} = y^L_{\alpha\beta} \overline{L}_\alpha E_\beta H + y^D_{\alpha\beta} \overline{Q}_\alpha D_\beta H + y^U_{\alpha\beta} \overline{Q}_\alpha U_\beta \widetilde{H}$$

$$L_{Higgs} = -V = m^2 H^\dagger H - \tfrac{\lambda}{2} (H^\dagger H)^2$$

$$\widetilde{H} = i\tau_2 H^\dagger$$

α,β=1,2,3 - generation index

# Fermion Masses in the SM

Direct mass terms are forbidden due to $SU(2)_L$ invariance !

Dirac Spinors left right Dirac conjugated Charge conjugated

$$\psi,\ \psi_L = \frac{1-\gamma^5}{2}\psi,\ \psi_R = \frac{1+\gamma^5}{2}\psi\ ,\ \overline{\psi} = \psi^+\gamma^0,\ \psi^c = C\gamma^0\psi = i\gamma^2\psi^*$$

Lorenz invariant Mass terms

$SU_L(2)$

$$\cancel{\overline{\psi}_L\psi_R} + \cancel{\overline{\psi}_R\psi_L} \qquad \overline{\psi}_L\psi_L = \overline{\psi}_R\psi_R = 0$$

SU(2) doublet SU(2) singlet

$SU_L(2)$ & $U_Y(1)$ $U_Y(1)$

$$\cancel{\overline{\psi}^c_L\psi_L + \overline{\psi}_L\psi^c_L} \qquad \cancel{\overline{\psi}^c_R\psi_R + \overline{\psi}_R\psi^c_R}$$

Unless Q=0, Y=0

$$\overline{\nu}^c_R\nu_R$$

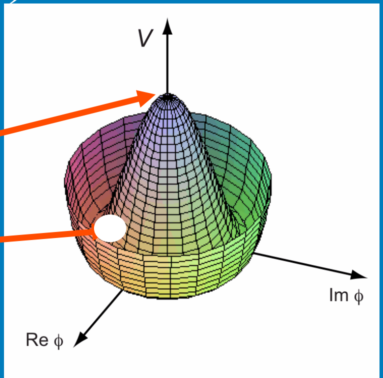
Majorana mass term

# Spontaneous Symmetry Breaking

$$SU_c(3)\otimes SU_L(2)\otimes U_Y(1)\rightarrow SU_c(3)\otimes U_{EM}(1)$$

Introduce a scalar field with quantum numbers: (1,2,1) $H=\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

With potential $$V=-m^2H^\dagger H+\tfrac{\lambda}{2}(H^\dagger H)^2$$




Unstable maximum

Stable minimum

At the minimum

v.e.v. scalar

$$H=\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}=\begin{pmatrix} H^+ \\ v+\dfrac{S+iP}{\sqrt{2}} \end{pmatrix}=\exp\left(i\frac{\vec{\xi}\vec{\sigma}}{2}\right)\begin{pmatrix} 0 \\ v+\dfrac{S}{\sqrt{2}} \end{pmatrix}$$

pseudoscalar

Gauge transformation

Higgs boson

$$H\longrightarrow H'=\exp\left(i\frac{\vec{\alpha}\vec{\sigma}}{2}\right)H\xrightarrow{(\vec{\alpha}=-\vec{\xi})}H'=\begin{pmatrix} 0 \\ v+\dfrac{\mathbf{h}}{\sqrt{2}} \end{pmatrix}$$

# The Higgs Mechanism

Q: What happens with missing d.o.f. (massless goldstone bosons P,H$^+$ or $\vec{\xi}$) ?

A: They become longitudinal d.o.f. of the gauge bosons $W_\mu^i$, i=1,2,3

Gauge transformation $\widehat{W}_\mu \to e^{i\alpha^a\sigma^a}\,\widehat{W}_\mu\, e^{-i\alpha^a\sigma^a} - \frac{1}{g}\underline{\partial_\mu\left(e^{i\alpha^a\sigma^a}\right)e^{-i\alpha^a\sigma^a}}$

$\alpha^a = -\xi^a$

Longitudinal components

Higgs field kinetic term $\left|D_\mu H\right|^2 = \left|\partial_\mu H - \frac{g}{2}\widehat{W}_\mu H - \frac{g'}{2}\widehat{B}_\mu H\right|^2 \longleftarrow H = \begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix}$

$$\to \tfrac{1}{4}(0\ \mathrm{v})\begin{pmatrix} \mathrm{g}W_\mu^3 + g'B_\mu & \sqrt{2}\mathrm{g}W_\mu^- \\ \sqrt{2}\mathrm{g}W_\mu^+ & -\mathrm{g}W_\mu^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} \mathrm{g}W_\mu^3 + g'B_\mu & \sqrt{2}\mathrm{g}W_\mu^- \\ \sqrt{2}\mathrm{g}W_\mu^+ & -\mathrm{g}W_\mu^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix}$$

$$\Rightarrow \frac{g^2}{2}\mathrm{v}^2 W_\mu^+ W_\mu^- + \frac{1}{4}\mathrm{v}^2(-gW_\mu^3 + g'B_\mu)^2$$

$$M_W^2 = \tfrac{1}{2}g^2\mathrm{v}^2$$
$$M_Z^2 = \tfrac{1}{2}(g^2 + \mathrm{g'}^2)\mathrm{v}^2$$

$$\tan\theta_W = g'/g$$
$$M_\gamma = 0$$

$$W_\mu^\pm = \frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}}$$
$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$$
$$\gamma_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

# The Higgs Boson and Fermion Masses

$$H = \begin{pmatrix} 0 \\ \mathrm{v} + \frac{h}{\sqrt{2}} \end{pmatrix} \Rightarrow V = -m^2 H^\dagger H + \tfrac{\lambda}{2}(H^\dagger H)^2$$

$$\Rightarrow V = -\frac{\lambda \mathrm{v}^4}{2} + \lambda \mathrm{v}^2 h^2 + \frac{\lambda \mathrm{v}}{\sqrt{2}} h^3 + \frac{\lambda}{8} h^4 \qquad \mathrm{v}^2 = m^2 / \lambda$$

$$\boxed{m_h = \sqrt{2} m = \sqrt{2\lambda}\,\mathrm{v}}$$

$$L_{Yukawa} = y^E_{\alpha\beta} \overline{L}_\alpha E_\beta H + y^D_{\alpha\beta} \overline{Q}_\alpha D_\beta H + y^U_{\alpha\beta} \overline{Q}_\alpha U_\beta \widetilde{H}$$

α, β =1,2,3 - generation index

Dirac fermion mass

$$\boxed{M_i^u = Diag(y^u_{\alpha\beta})\mathrm{v}, \quad M_i^d = Diag(y^d_{\alpha\beta})\mathrm{v}, \quad M_i^l = Diag(y^l_{\alpha\beta})\mathrm{v}}$$

$$y^N_{\alpha\beta} \overline{L}_\alpha N_\beta \widetilde{H} \to M_i^\nu = Diag(y^N_{\alpha\beta})\mathrm{v}$$ Dirac neutrino mass

# Quark/Lepton Mixing

- The mass matrix is non-diagonal in generation space
- It can be diagonalized by field rotation Q -> Q'= V Q

$$\bar{U} M_U U -> \bar{U}' V_U^+ M_U V_U U' = \bar{U}' M_U^{Diag} U'$$
$$\bar{D} M_D D -> \bar{D}' V_D^+ M_D V_D D' = \bar{D}' M_D^{Diag} D'$$

- Neutral Current: $\bar{U} Z_\mu U -> \bar{U}' V_U^+ Z_\mu V_U U' = \bar{U}' Z_\mu U' \; V_U^+ V_U = \bar{U}' Z_\mu U'$

- Charged Current $\bar{U} W_\mu D -> \bar{U}' V_U^+ W_\mu V_D D = \bar{U}' W_\mu V_U^+ V_D D'$

Cabibbo-Kobayashi-Maskawa mixing matrix $K = V_U^+ V_D$

The (only) source of flavour mixing in the SM

Unitarity: $K^+ K = 1$

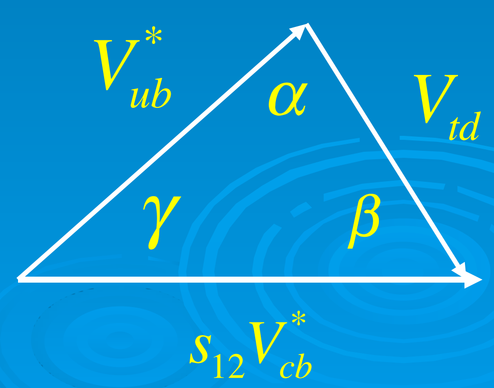
# CKM Matrix and Unitarity Triangle

$$K = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

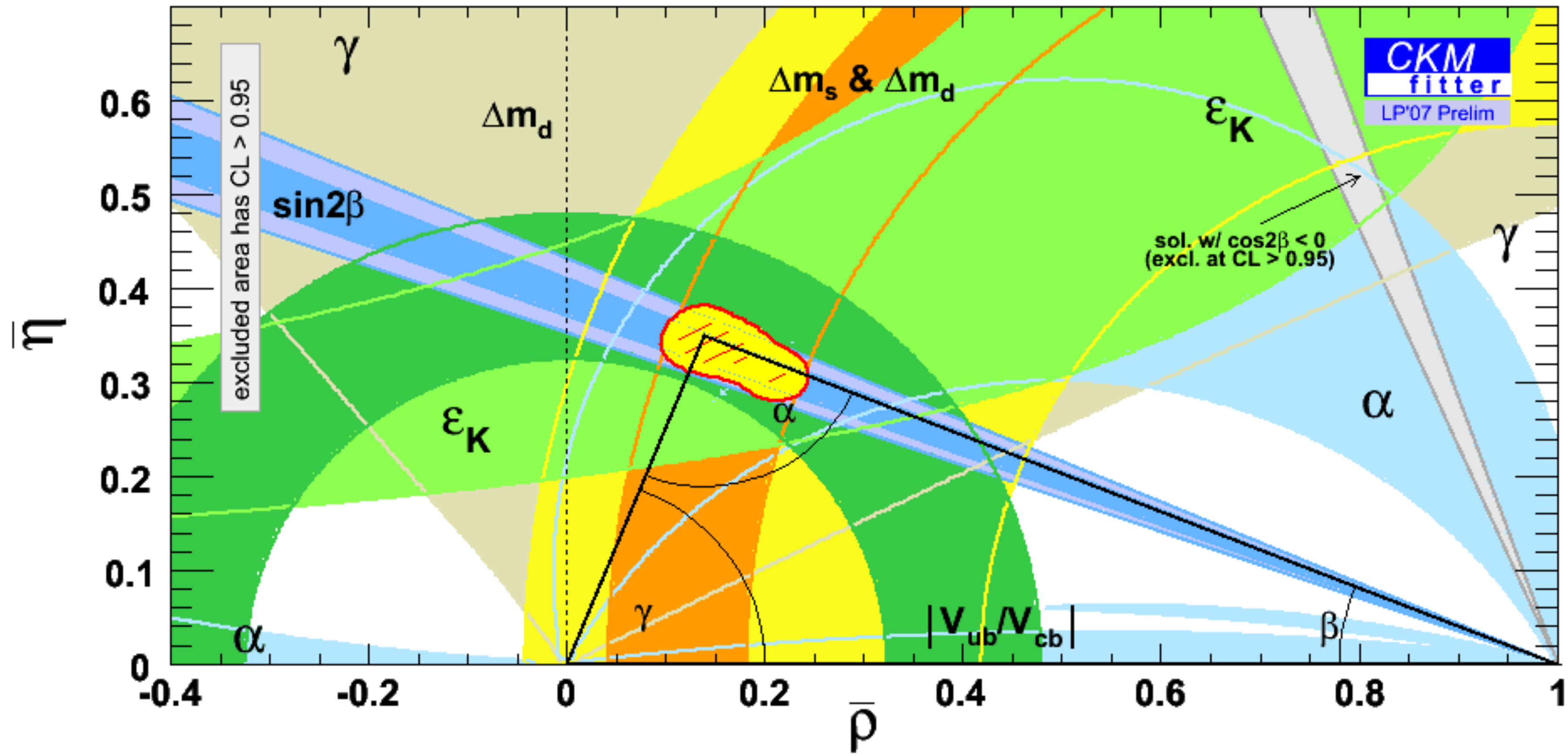
Two important properties

1. CP-violation due to a complex phase !
2. Unitarity triangle

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$
$$\Rightarrow \quad V_{ub}^{*} + V_{td} = s_{12}V_{cb}^{*}$$



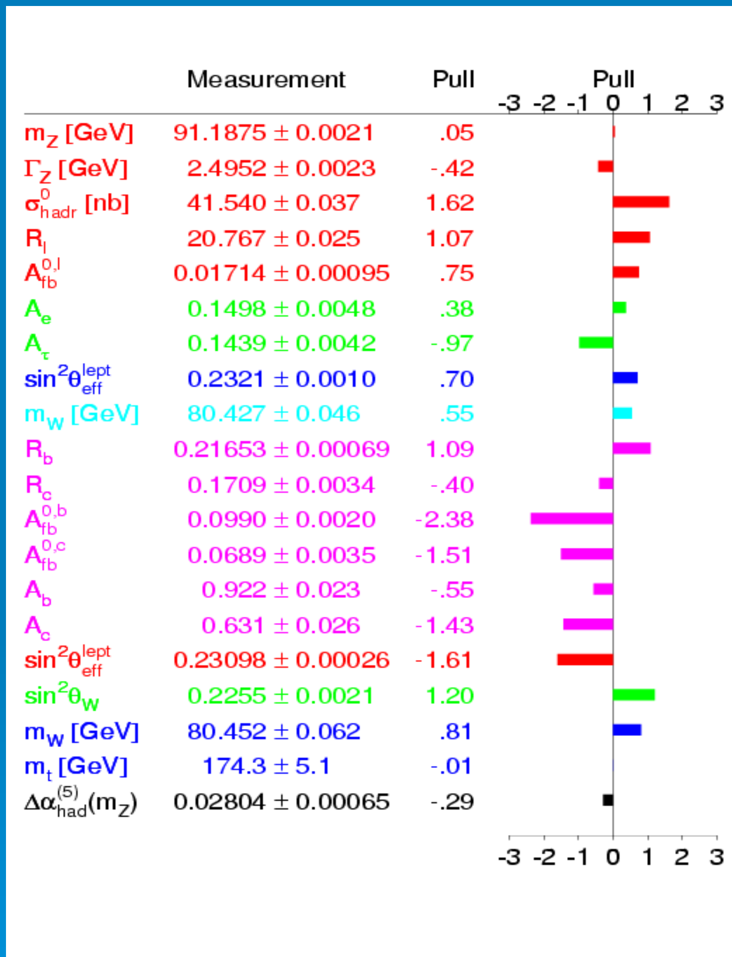
# The Unitarity Triangle: all constraints



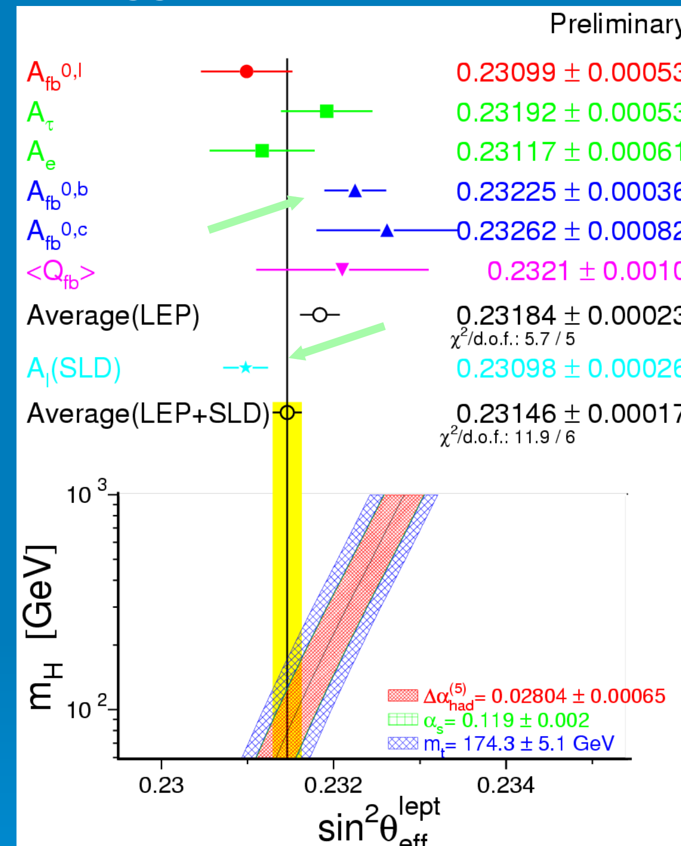

A consistent picture across a huge array of measurements

# Comparison with Experiment

Global Fit to Data



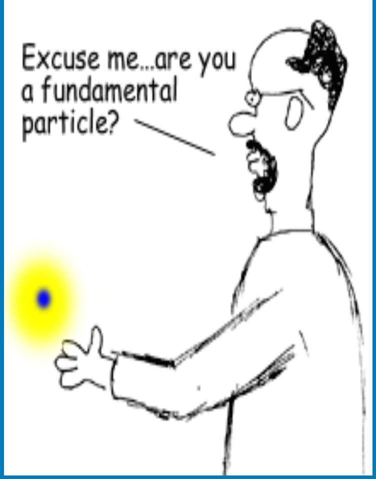

Higgs Mass Constraint




Remarkable agreement of ALL the data with the SM predictions - precision tests of radiative corrections and the SM

Though the values of sin w extracted from different experiments are in good agreement, two most precise measurements from hadron and lepton asymmetries disagree by 3

# The SM and Beyond




The problems of the SM:

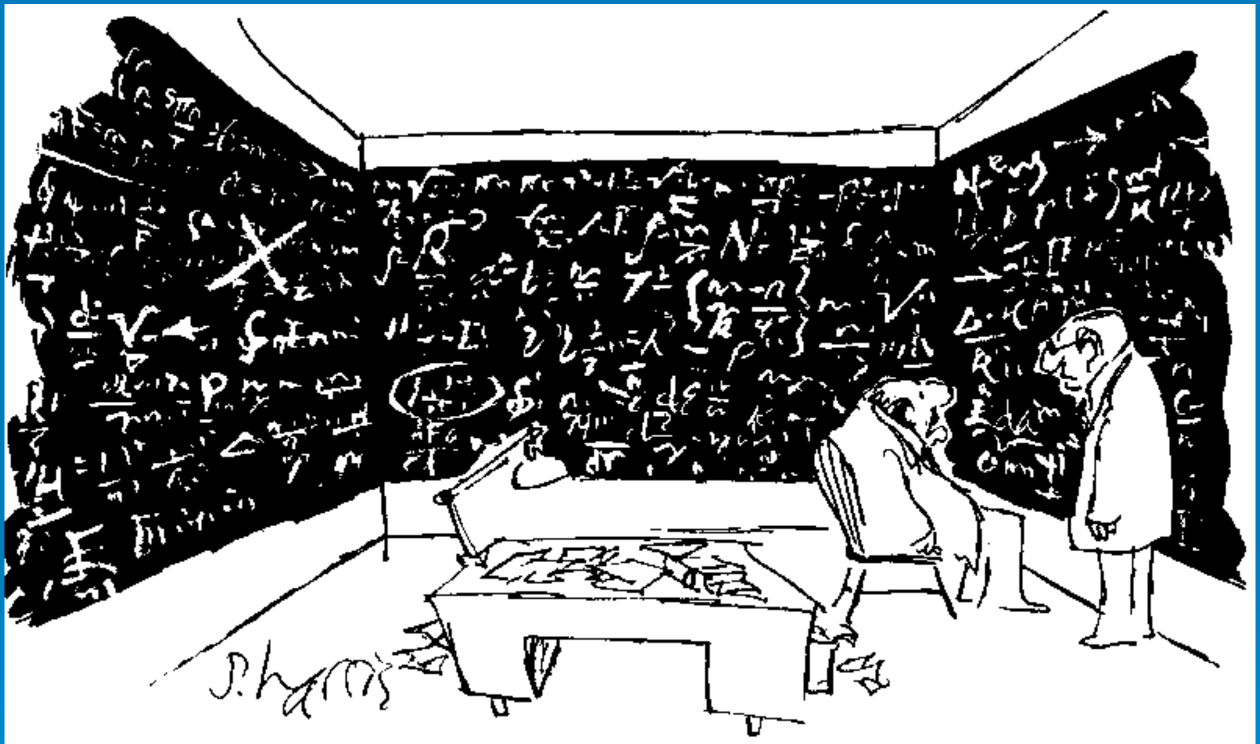
- Inconsistency at high energies due to Landau poles
- Large number of free parameters
- Still unclear mechanism of EW … breaking
- CP-violation is not un…
- The origin of the m… unclear
- Flavour mixing and … ber of generations is arbitrary
- Formal unification o… trong and electroweak interactions

Where is the Dark matter?

The way beyond the SM:

- The SAME fields with NEW interactions and NEW fields ⇨ GUT, SUSY, String, ED
- NEW fields with NEW interactions ⇨ Compositeness, Technicolour, preons

# We like elegant solutions



"Whatever happened to *elegant* solutions?"